AB Calculus
Ch. 1 Notes

Name Key (2014)

Day 1

Linear Equations:

slope-intercept	general	point-slope
$y = mx + b$	$Ax + By = C$	$y - y_1 = m(x - x_1)$

we use this a ton in calculus!

Example: Write the equation of the line thru (7, 2), (-2, 3).

$3 - 2 = m(-2 - 7)$

$1 = m(-9)$

$-\frac{1}{9} = m$

$\boxed{y - 2 = -\frac{1}{9}(x - 7)}$

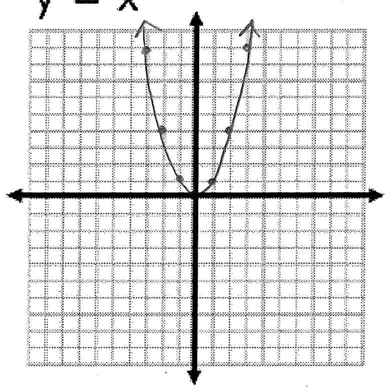
Example: Write the equation of a line thru (7, 2) and perpendicular to the line in previous example.

$\boxed{y - 2 = 9(x - 7)}$

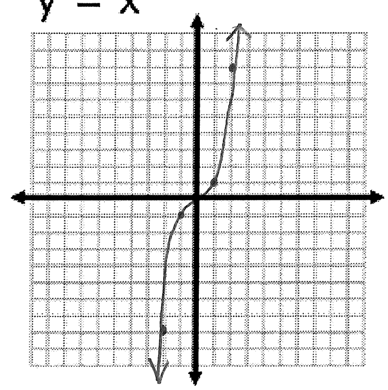
*unless necessary, leave eqn. of line in point-slope form!

Parent functions:

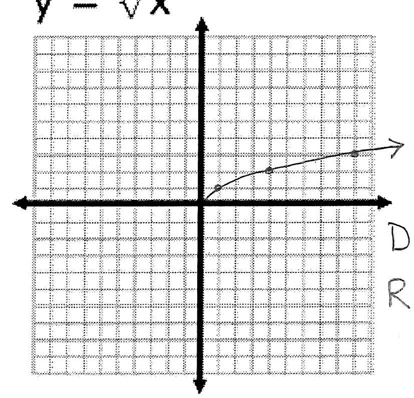
$y = x^2$



$y = x^3$



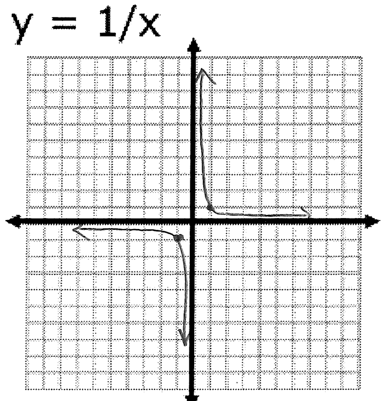
$y = \sqrt{x}$



D: $[0, \infty)$

R: $[0, \infty)$

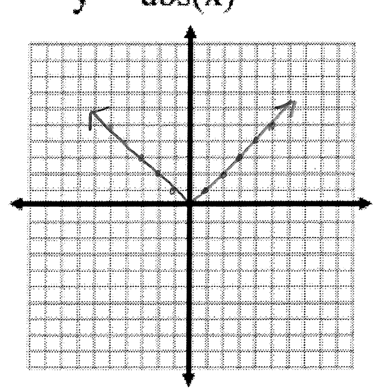
$y = 1/x$



D: $(-\infty, 0), (0, \infty)$

R: $(-\infty, 0), (0, \infty)$

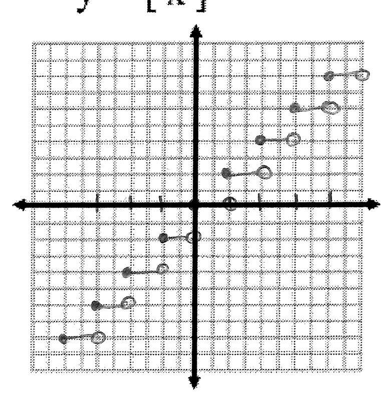
y = abs(x)



D: $(-\infty, \infty)$

R: $[0, \infty)$

y = [x]



Domain: x values

ex. $-1 < x \leq 4$

$(-1, 4] \leftarrow$ new notation $\longrightarrow [0, \infty)$

Range: y values

ex. $y \geq 0$

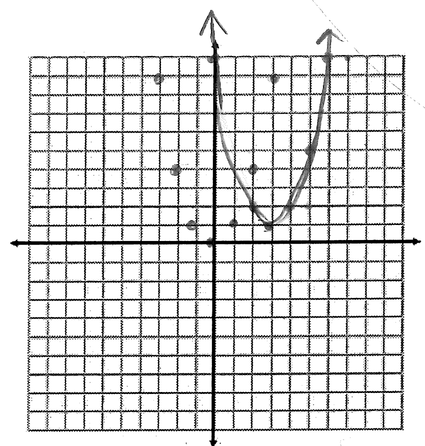
Example: Graph, then state the domain and range for

$$y = (x-3)^2 + 1$$

shift right 3

shift up 1

parent function: $y = x^2$



D: $-\infty \leq x \leq \infty$ or $(-\infty, \infty)$

R: $y \geq 1$ or $[1, \infty)$

Example: State the domain and range for $y = \sqrt{x-5}$

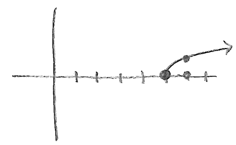
$x - 5 \geq 0$

$x \geq 5$

$y \geq 0$

D: $[5, \infty)$

R: $[0, \infty)$



Example: State the domain and range for $y = \frac{1}{x+2} - 3$

left 2

down 3

$x \neq 2$

$\frac{1}{x+2} \neq 0 \rightarrow \frac{1}{x+2} - 3 \neq -3$

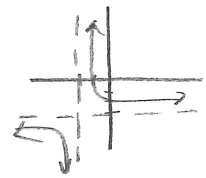
$y \neq -3$

D: x = all reals

$x \neq 2$

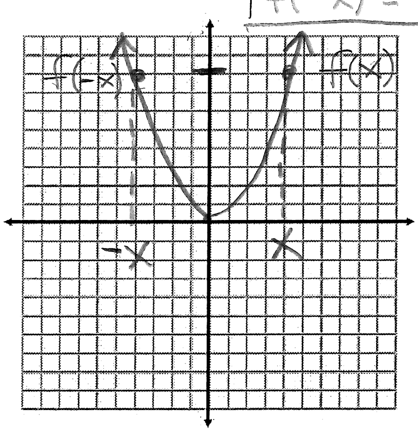
R: y = all reals

$y \neq -3$



Even functions: symmetric w.r.t. y-axis

$\boxed{f(-x) = f(x)}$



ex:

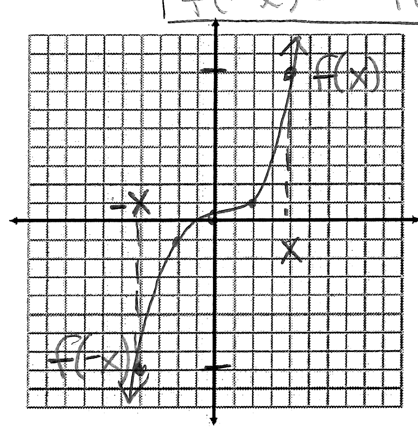
$y = x^2$

$y = |x|$

$y = \cos x$

Odd functions: symmetric w.r.t. origin

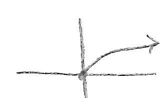
$\boxed{f(-x) = -f(x)}$



ex:

$y = x^3$

$y = \sin x$



$y = \sqrt{x}$ neither odd nor even

Example: Verify whether $y = 4x^3 - x$ is even, odd, or neither.

$f(x) = 4x^3 - x$

$f(-x) = 4(-x)^3 - (-x)$

$= -4x^3 + x = -f(x)$

$f(x) = -f(x)$

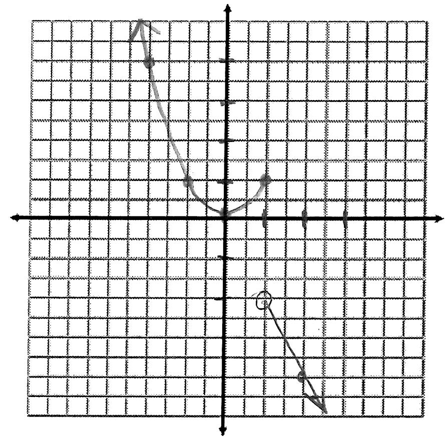
Piecewise functions:

Example: Graph, then state domain and range.

$$f(x)=\begin{cases} x^2, & x \le 1 \\ -2x, & x > 1 \end{cases}$$

D: $(-\infty, \infty)$

R: $(-\infty, -2)$, $[0, \infty)$

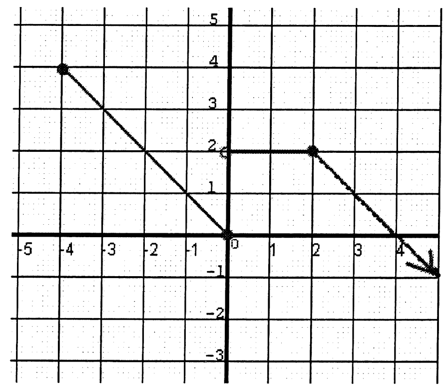


Example: Write the equation of the piecewise function and state the domain and range.

$$f(x)=\begin{cases} -x & \text{for } -4 \le x \le 0 \\ 2 & \text{for } 0 < x \le 2 \\ -x+4 & \text{for } x > 2 \end{cases}$$

D: $[-4, \infty)$

R: $(-\infty, 4]$



Day 2

Rational Expression: $\dfrac{\text{function}}{\text{function}}$

Example: Simplify. → get common denominator (factor first), combine

$$\frac{d-4}{d^2+2d-8} - \frac{d+2}{d^2-16}$$

$$= \frac{d-4}{(d-2)(d+4)} - \frac{d+2}{(d+4)(d-4)}$$

$$= \frac{(d-4)(d-4) - (d+2)(d-2)}{(d-2)(d+4)(d-4)}$$

$$= \frac{d^2-8d+16-(d^2-4)}{(d-2)(d+4)(d-4)}$$

$$= \frac{-8d+20}{(d-2)(d+4)(d-4)} = \boxed{\frac{-4(2d+5)}{(d-2)(d+4)(d-4)}}$$

$$\frac{m^2+n^2}{m^2-n^2} + \frac{m}{n-m} + \frac{n}{m+n}$$

$$= \frac{m^2+n^2}{(m+n)(m-n)} + \frac{m}{-(m-n)} + \frac{n}{m+n}$$

$$= \frac{m^2+n^2 + -m(m+n) + n(m-n)}{(m+n)(m-n)}$$

$$= \frac{m^2+n^2-m^2-mn+mn-n^2}{(m+n)(m-n)}$$

$$= \boxed{0}$$

$$\frac{y}{y}\cdot\frac{\frac{1}{x}-\frac{1}{y}}{1+\frac{1}{x}}\cdot\frac{x}{x} \qquad \frac{\frac{y-x}{xy}}{\frac{x+1}{x}}$$

$$=\frac{y-x}{xy}\cdot\frac{x}{x+1}=\boxed{\frac{y-x}{y(x+1)}}$$

$$\frac{b-5}{b-5}\cdot\frac{\frac{1}{b+2}+\frac{1}{b-5}}{\frac{2b^2-b-3}{b^2-3b-10}}\cdot\frac{b+2}{b+2}=\frac{\frac{b-5+b+2}{(b+2)(b-5)}}{\frac{(2b-3)(b+1)}{(b-5)(b+2)}}$$

$$=\frac{2b-3}{(b+2)(b-5)}\cdot\frac{(b-5)(b+2)}{(2b-3)(b+1)}$$

$$=\boxed{\frac{1}{b+1}}$$

Example: Solve.

$$t\left(t+\frac{12}{t}\right)=(8)t \qquad \underline{D: t\neq 0}$$

$$t^2+12=8t$$

$$t^2-8t+12=0$$

$$(t-6)(t-2)=0$$

$$\boxed{t=2,6}$$

check: $2+\frac{12}{2}\overset{?}{=}8$ ✓

$6+\frac{12}{6}\overset{?}{=}8$ ✓

$$r+\frac{r^2-5}{r^2-1}=\frac{r^2+r+2}{r+1} \qquad \underline{D: r\neq 1,-1}$$

$$\frac{r(r+1)(r-1)}{(r+1)(r-1)}+\frac{r^2-5}{(r+1)(r-1)}=\frac{(r^2+r+2)(r-1)}{(r+1)(r-1)}$$

$$\frac{r(r^2-1)+r^2-5}{(r+1)(r-1)}=\frac{r^3+r^2+2r-r^2-r-2}{(r+1)(r-1)}$$

$$r^3-r+r^2-5=r^3+r-2$$

$$r^2-2r-3=0$$

$$(r+1)(r-3)=0$$

$$r=3,-1 \qquad \boxed{r=3}$$

not in domain

$$\frac{4}{z-2}-\frac{z+6}{z+1}=1 \qquad D: z\neq 2,-1$$

$$\frac{4(z+1)-(z+6)(z-2)}{(z-2)(z+1)}=\frac{(z-2)(z+1)}{(z-2)(z+1)}$$

$$4z+4-(z^2+4z-12)=z^2-z-2$$

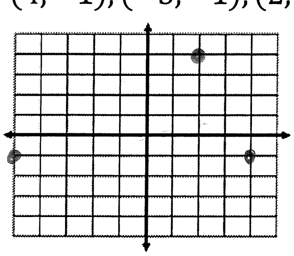
$$-z^2+16=z^2-z-2$$

$$0=2z^2-z-18$$

$$z=\frac{1\pm\sqrt{1-4(2)(-18)}}{2(2)}=\boxed{\frac{1\pm\sqrt{145}}{4}}$$

<u>Function</u>: 1 y-value for each x-value

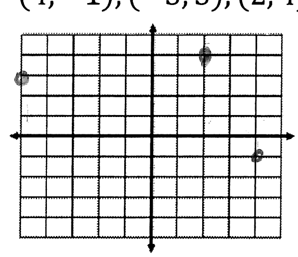
$(4,-1), (-5,-1), (2,4)$



passes vertical line test (VLT)

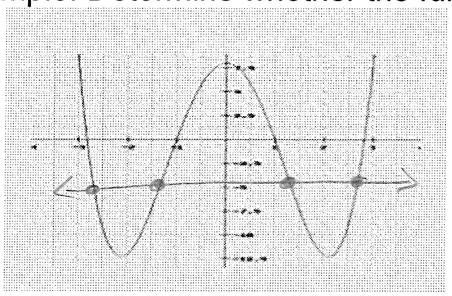
<u>1 to 1 Function</u>: Function w/ 1 x-value for each y-value

$(4,-1), (-5,3), (2,4)$

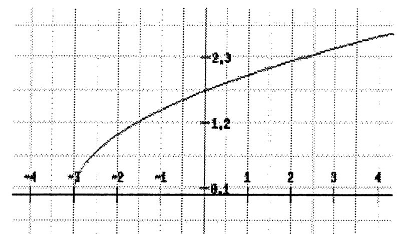


passes <u>both</u> VLT and HLT

Example: Determine whether the function is one-to-one.



No, fails HLT



Yes, passes both VLT & HLT

Composition of Functions: Function inside a function

Example: Given $f(x) = x^2 + x$ and $g(x) = \frac{2}{x+3}$, find:

$(f \circ g)(3) = f(g(3))$

$g(3) = \frac{2}{3+3} = \frac{2}{6} = \frac{1}{3}$

$f(g(3)) = f(\frac{1}{3}) = (\frac{1}{3})^2 + \frac{1}{3} = \frac{1}{9} + \frac{1}{3}$

$= \frac{1}{9} + \frac{3}{9} = \boxed{4/9}$

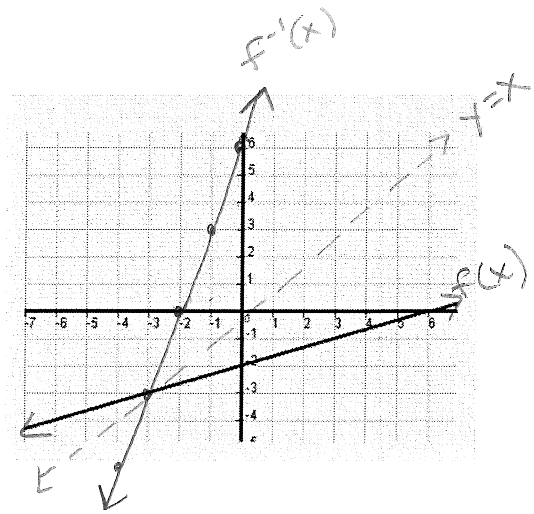
$(f \circ f)(x) = f(f(x))$

$= (x^2+x)^2 + (x^2+x)$

$= x^4 + 2x^3 + x^2 + x^2 + x$

$= \boxed{x^4 + 2x^3 + 2x^2 + x}$

Inverses:
- swap x's and y's, solve for y
- notation:
  $f^{-1}(x)$ = inverse of $f(x)$
- f and f^{-1} are reflections over $y=x$ and slopes are reciprocals.



Example: Find inverse of $f(x) = \frac{x}{3} - 2$ $\quad y = \frac{x}{3} - 2$

$x = \frac{y}{3} - 2$

$x + 2 = \frac{y}{3}$

$3x + 6 = y$

$\boxed{y = 3x + 6}$ $f^{-1}(x)$

Theorem: Two functions are inverses if and only if $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Example: Verify that $f(x) = \frac{x}{3} - 2$ and $f^{-1}(x) = 3x + 6$ are inverses.

$f(f^{-1}(x)) = \frac{3x+6}{3} - 2$

$= \frac{\cancel{3}(x+2)}{\cancel{3}} - 2$

$= x + 2 - 2$

$= x$

$f^{-1}(f(x)) = 3(\frac{x}{3} - 2) + 6$

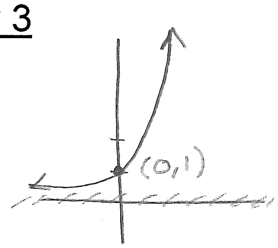
$= x - 6 + 6$

$= x$

Yes, inverses

Day 3

Exponential function: $y = a^x$



← asymptote $y=0$

D: $(-\infty, \infty)$
R: $(0, \infty)$

Example: Graph and state domain, range, and zeros. $y = 2^x - \frac{3}{2}$

↑ shift down $\frac{3}{2}$

by hand:

$0 = 2^x - \frac{3}{2}$

$\frac{3}{2} = 2^x$

$\log \frac{3}{2} = \log 2^x = x \log 2$

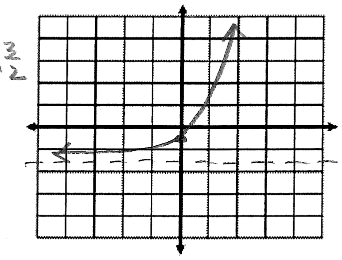
$\frac{\log \frac{3}{2}}{\log 2} = x$

$x \approx .585$

on calc:

graph $y_1 = 2^x - \frac{3}{2}$

calc → zero



D: $(-\infty, \infty)$
R: $(-\frac{3}{2}, \infty)$

Example: How much time does it take to triple your savings if the interest is compounded continuously at 5.75%?

$r = .0575$

$A = Pe^{rt}$

A = final amt
P = start amt (principal)
r = rate (as decimal)
t = time
$e \approx 2.718$

$3 = 1e^{.0575t}$

$3 = e^{.0575t}$

$\ln 3 = \ln e^{.0575t} = .0575t(\ln e)$ $\ln e = 1$

$\ln 3 = .0575t$

$t = \frac{\ln 3}{.0575} \approx \boxed{19.106 \text{ years}}$

Example: The half-life of Carbon-14 is 5730 years. If 10 grams were present originally, how much will be left after 2000 years?

half-life: amt of time needed for ½ of material to decay

10 grams: y_0

$y = y_0 e^{kt}$ ← same as $A = Pe^{rt}$

y = final amt
y_0 = start amt
k = rate of decay
t = time

$5 = 10e^{k \cdot 5730}$

$\frac{1}{2} = e^{k \cdot 5730}$

$\ln \frac{1}{2} = \ln e^{k \cdot 5730} = k \cdot 5730$

$k = \frac{\ln \frac{1}{2}}{5730}$ $\qquad k = -.00012097$

store in calc. mem

used stored value

$y = 10e^{(-.00012097)(2000)} = \boxed{7.851 \text{ g.}}$

* properties of logs: p.41 in book

Logarithms:

$\log_b a = x$

$\Updownarrow$ equivalent

$b^x = a$

Example:

- Find inverse of $y = 10^x$. $\xrightarrow{\text{inverse}}$ $y = \log x$
- Graph both functions. $\rightarrow$ reflection over $y = x$
- Find domain and range for both functions.

$x = 10^y$

$\log x = \log 10^y$

$\log x = y(\log 10)$ $\rightarrow = 1$

$\log x = y$

$y = 10^x$

x	y
-2	1/100
-1	1/10
0	1
1	10
2	100

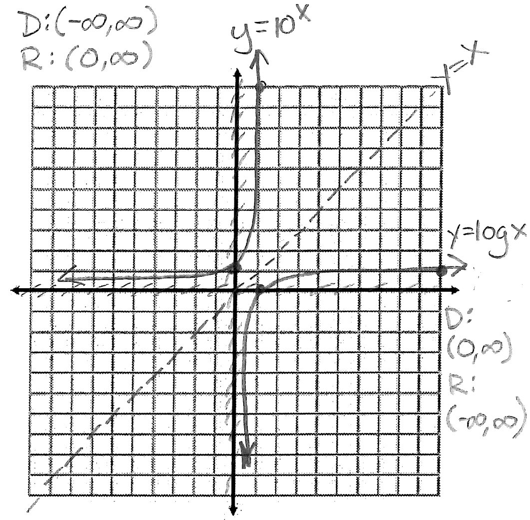
HA: x axis

$y = \log x$

x	y
1/100	-2
1/10	-1
1	0
10	1
100	2

V.A.: y axis

D: $(-\infty, \infty)$
R: $(0, \infty)$



D: $(0, \infty)$
R: $(-\infty, \infty)$

Example: Find domain and range of $f(x) = ln(x - 2)$.

$\downarrow$ shift right 2

$y = \log_e(x-2)$

equivalen

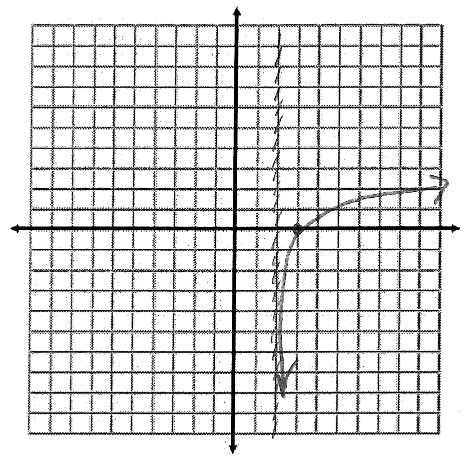
$\underbrace{e^y}_{\text{always} > 0} = x - 2$

so $x - 2 > 0$

$x > 2$

D: $(2, \infty)$

R: $(-\infty, \infty)$



Example: Solve $e^{x+2} = 3$ algebraically, then support graphically.

$e^{x+2} = 3$

$\ln e^{x+2} = \ln 3$

$(x+2) \cdot \ln e = \ln 3$

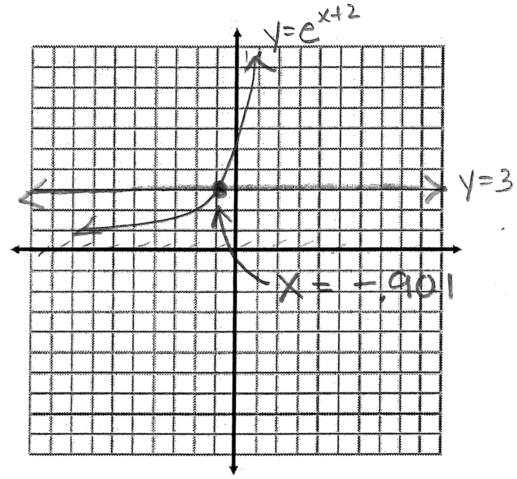
$x + 2 = \ln 3$

$\boxed{x = \ln 3 - 2 \approx -.901}$

1) graph: $y_1 = e^{x+2}$

$y_2 = 3$

2) find intersection (calc $\rightarrow$ intersect)



Example: Given $lny - 5 = 7 + lnx$, solve for y.

$\ln y = 12 + \ln x$

$e^{\ln y} = e^{(12 + \ln x)}$ $\leftarrow$ "exponentiate" both sides

$y = e^{12} \cdot e^{\ln x}$

$y = e^{12} \cdot x$

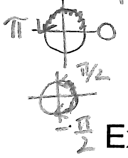
recall:

$x^2 \cdot x^3 = x^{2+3}$

so

$e^{x+y} = e^x \cdot e^y$

Principal values, unless stated otherwise



$\theta = \cos^{-1}x, \sec^{-1}x, \cot^{-1}x$

$\theta = \sin^{-1}x, \csc^{-1}x, \tan^{-1}x$

TRIGONOMETRY

SOH - CAH - TOA

$(x, y) = (\cos\theta, \sin\theta)$

Day 4

Example:

$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$ ← angle ← ratio: $\frac{opp}{hyp}$

angle (backwards) → $\tan\left(\frac{-3\pi}{4}\right) = 1$ ← ratio: $\frac{opp}{adj}$

$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$ ← ratio: $\frac{hyp}{adj}$

arccos → find ∠

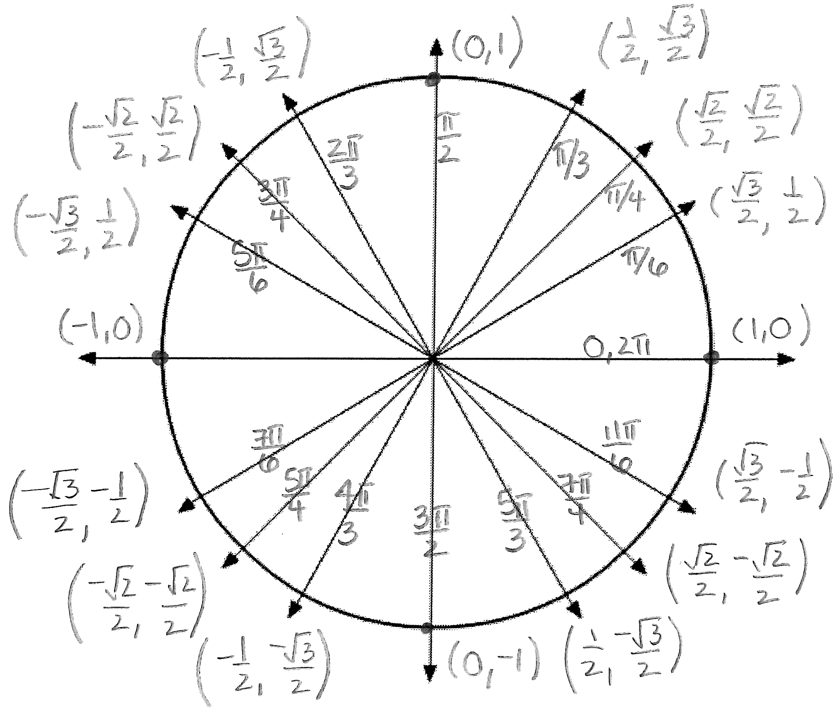
$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ ← principal ∠

ratio: $\frac{adj}{hyp}$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$

arcsin

ratio:



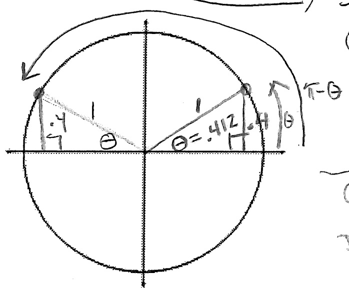
Example: Solve for θ.

$\sin\theta = 0.4, \; 0 \le \theta < 2\pi$ (angle; ratio)

same as

$\theta = \sin^{-1}(.4)$

angle; ratio opp/hyp



Calculator gives principal value:

$\boxed{\theta = .412 \text{ radians}}$ (23.578°)

and

$\theta = \pi - .412$

$\boxed{\theta = 2.730 \text{ rad.}}$

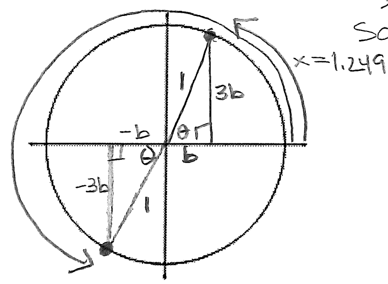
156.422°

$\tan x = 3, \; 2\pi \le x < 4\pi$

≈6.283 ≈12.566

Same as

$x = \tan^{-1}3$ ← ratio opp/adj



Calculator gives

$x = 1.249$ rad

(71.565°)

not in domain

$\theta = 1.249 + 2\pi = \boxed{7.532}$ 332°

and

$\theta = 7.532 + \pi = \boxed{10.674}$ 512°

Example: Determine the 6 trig. ratios for θ with terminal side through $(-3, -4)$.

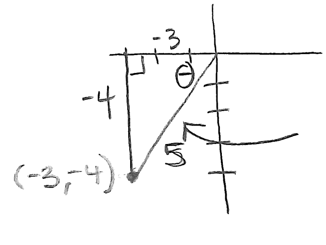
reciprocals

$\sin\theta = \frac{-4}{5}$ $\quad \csc\theta = \frac{-5}{4}$

$\cos\theta = -\frac{3}{5}$ $\quad \sec\theta = \frac{-5}{3}$

$\tan\theta = \frac{4}{3}$ $\quad \cot\theta = \frac{3}{4}$

draw Δ on grid.



$3^2 + 4^2 = c^2$

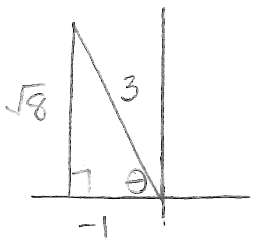
$9 + 16 = c^2$

$25 = c^2$

Example: Determine the 6 trig. ratios if $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

angle → ratio: $\frac{adj}{hyp}$

draw Δ!



$1^2 + b^2 = 3^2$
$1 + b^2 = 9$
$b^2 = 8$

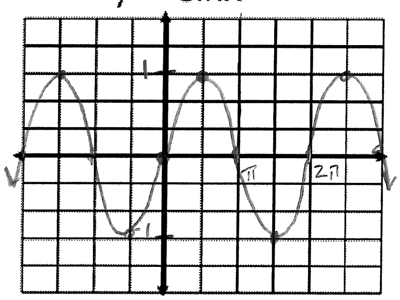
$\sin\theta = \frac{\sqrt{8}}{3}$ $\csc\theta = \frac{3}{\sqrt{8}}$

$\cos\theta = \frac{-1}{3}$ $\sec\theta = -3$

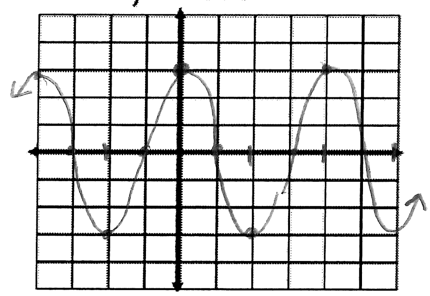
$\tan\theta = -\sqrt{8}$ $\cot\theta = \frac{-1}{\sqrt{8}}$

Graphs of trigonometric functions:

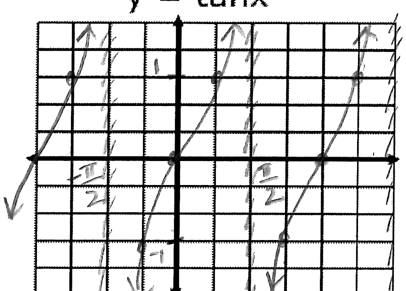
y = sinx



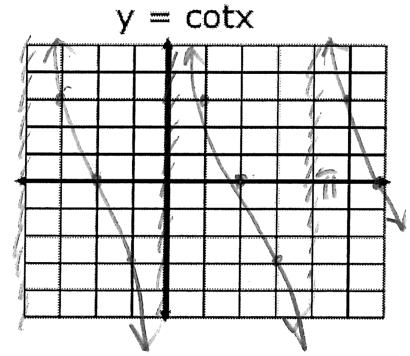
y = cosx



y = tanx



y = cotx



amplitude = |A|

$y = A\cos(k\theta + c) + d$ ← vertical shift

phase shift: $ps = -c/k$

period: $per = \frac{2\pi}{|k|}$ ($per = \frac{\pi}{k}$ for tan or cot)

Example: Graph $y = 4\cos\left(\frac{x}{2} - \pi\right) + 1$

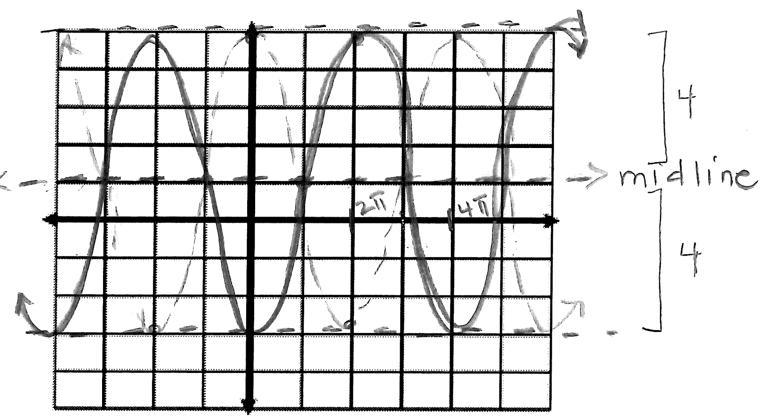
$k = \frac{1}{2}$ $c = -\pi$

② mark vert. bounds → $A = 4$

$p.s. = \frac{\pi}{1/2} = 2\pi$ (right)

$per = \frac{2\pi}{1/2} = 4\pi$

$V.S. = 1$



<- -> midline

4

4

① start w/ this: draw "midline"

③ sketch ④ label scale according to period ⑤ shift